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The Necessary Number of Elements in a Directional Ring Aerial

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An investigation is made concerning the dependence of the array characteristic of a directional ring aerial on its number of elements. It is shown that an odd number of elements is more favorable than an even number in approximating the array characteristic of a similar ring aerial with infinitely many elements. It was previously shown by Page that a similar statement applies to an azimuthly omnidirectional, fading-reducing, concentric ring aerial; this result is contained as a special case in this investigation. The present paper deals especially with the case in which the principal direction of the antenna array is horizontal; the theory for this case is illustrated by a numerical example.

INTRODUCTION

A RING aerial, i.e., an array of s similar and similarly oriented antennas placed equidistantly along a circle, is for sufficiently large s approximately rotationally symmetrical. (For $s=\infty$ the antenna system has complete rotational symmetry.) This approximate rotational symmetry of the ring aerial is used in different ways.

In all the cases that will be dealt with here, the numerical value of the current is assumed to be the same in all elements of the array. Also for the matter of convenience in describing the field, the plane of the circle is assumed horizontal.

Chireix[1] has shown that by phasing the elements progressively so that the phase increases $2\pi n$, where n is an integer, when passing once around the circle, an antenna system with reduced radiation at high elevation angles is obtained. Since the ring aerial obtained in this way is approximately azimuthly omnidirectional for s sufficiently large, it will be useful as a fading reducing antenna for broadcasting purposes. A similar array but with several concentric rings was later independently proposed by Hansen and Woodyard[2,*] and investigated further by Hansen and Hollingsworth.[3,*] Another type of an omnidirectional antenna with reduced radiation at high elevation angles may be obtained by driving a ring aerial with all currents in the same phase and adding at the center of the ring an antenna carrying a current in phase opposition to the currents in the outer antennas and with an amplitude somewhat less than the sum of the current amplitudes in all of the outer antennas. This array may be considered a special case of one of the arrays investigated by Hansen and Woodyard.[2] The application of this ring aerial as a fading reducing antenna is described by Böhm[4,5] and by Harbich and Hahnemann.[6] The radiation resistance of the above mentioned antenna arrays was calculated by Page.[7] The discrepancy between the array characteristics of ring aerials of the type referred to above with a finite number of elements and the corresponding aerials with infinitely many elements was touched on already by Chireix,[1] by Hansen and Woodyard,[2] and by Hansen and Hollingsworth[3], and was recently investigated in detail by Page.[8] Page showed that in the case of a concentric ring aerial, an odd number of antennas in the ring gives a better approximation of the array characteristic to the array characteristic in the ideal case of infinitely many antennas than an even number of antennas in the ring.

[1] H. Chireix, l'Onde Électr. **15**, 440–456 (1936).
[2] W. W. Hansen and J. R. Woodyard, Proc. Inst. Radio Engrs. **26**, 333–345 (1938).
* I am indebted to Dr. J. Epstein of the RCA Laboratories Division, Radio Corporation of America, Princeton, New Jersey, for drawing my attention to these papers.
[3] W. W. Hansen and L. M. Hollingsworth, Proc. Inst. Radio Engrs. **27**, 137–143 (1939).
[4] O. Böhm, Telefunken Zeitung **13**, 60, 21–26 (1932).
[5] O. Böhm, Hochfreq. techn. u. Elektroak. **42**, 137–145 (1933).
[6] H. Harbich and W. Hahnemann, Elektr. Nachr. Technik **9**, 361–376 (1932).
[7] H. Page, Wireless Engineer **25**, 102–109 (1948).
[8] H. Page, Wireless Engineer **25**, 308–315 (1948).

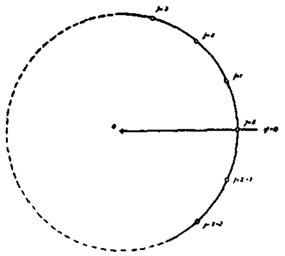

FIG. 1. Ring aerial with s elements.

Stenzel[9] has made an application of the approximate rotational symmetry of a ring aerial quite different from the above mentioned. By giving the currents of the antennas in the ring a phase distribution such that there is constructive interference of the waves emitted in an arbitrary direction in space, hereafter called the principal direction, an array characteristic is obtained having a principal lobe in this direction. Stenzel considers especially the case where the principal direction is horizontal. If the principal direction is rotated in the horizontal plane, the horizontal diagram of the array characteristic will also rotate and, as a consequence of the approximate rotational symmetry of the ring aerial, with its shape almost unchanged. For the ring aerial mentioned here, Stenzel has investigated the discrepancy between the array characteristic in the case of a finite, even number of elements and the characteristic in the case of infinitely many elements, apparently assuming that there is no essential difference between the case of an even number of elements and that of an odd number of elements. This assumption is explicitly stated by Brückmann[10] in his textbook on antennas in the chapter dealing with Stenzel's theory. In the present paper it will be investigated whether the array characteristic of the directional ring aerial with an even number of antennas differs essentially from the characteristic of the ring aerial with an odd number of elements as was the case with the ring aerial investigated by Page.[8] This investigation is based on Stenzel's as well as on Page's theory. For the sake of completeness those parts of Stenzel's and Page's calculations that are used in the present investigation are included in this paper.

THE ARRAY CHARACTERISTIC FOR A RING AERIAL WITH AN ARBITRARY PRINCIPAL DIRECTION

Let s identical and identically oriented antennas be placed equidistantly along a circle with center O and radius ρ as shown in Fig. 1. A polar coordinate system (r,θ,φ) is introduced with its center at O, with its axis $\theta=0$ perpendicular to the plane of the ring aerial (the horizontal plane) and with the plane $\varphi=0$ passing through antenna s. In this coordinate system the azimuth, β_j, of antenna j is expressed by the equation

$$\beta_j = j2\pi/s.$$

The currents, I_j, in the s antennas are assumed to have the same numerical value, but different phases. Using as the time factor $e^{-i\omega t}$, we express I_j as follows

$$I_j = I_0 e^{i\delta_j}.$$

The array characteristic of the ring aerial with the center O as reference point and with the current sI_0 as reference current is then expressed in the following way

$$G=\frac{1}{s}\sum_{j=1}^{s} e^{i[\delta_j - k\rho\sin\theta\cos(\varphi-\beta_j)]},$$

where k denotes the specific transmission coefficient of the medium surrounding the antenna array.

The currents in the antennas are now given such phases, according to Stenzel's[9] proposal, that the waves emitted in an arbitrarily chosen direction (θ_0, φ_0), the principal direction, are in phase. This is obtained by choosing

$$\delta_j = k\rho\sin\theta_0\cos(\varphi_0-\beta_j).$$

With this choice of the phases δ_j the expression for the array characteristic becomes

$$G=\frac{1}{s}\sum_{j=1}^{s} e^{ik\rho[\sin\theta_0\cos(\varphi_0-\beta_j)-\sin\theta\cos(\varphi-\beta_j)]}.$$

Following Stenzel we now define an angle ξ by the equation

$$\cos\xi = \frac{\sin\theta\cos\varphi-\sin\theta_0\cos\varphi_0}{[(\sin\theta\cos\varphi-\sin\theta_0\cos\varphi_0)^2+(\sin\theta\sin\varphi-\sin\theta_0\sin\varphi_0)^2]^{\frac{1}{2}}}.$$

Further introducing

$$\rho' = \rho[(\sin\theta\cos\varphi-\sin\theta_0\cos\varphi_0)^2 + (\sin\theta\sin\varphi-\sin\theta_0\sin\varphi_0)^2]^{\frac{1}{2}},$$

we may now express the array characteristic in the following way

$$G=\frac{1}{s}\sum_{j=1}^{s} e^{-ik\rho'\cos(\xi-\beta_j)}.$$

Let us first consider the case where the number of antennas is infinitely large. Introducing in the expression for G

$$s\rightarrow\infty,$$

$$\beta_j\rightarrow\beta,$$

$$\Delta\beta_j = 2\pi/s\rightarrow d\beta,$$

the sum occurring in this expression is converted into

[9] H. Stenzel, Elektr. Nachr. Technik **6**, 165–181 (1929).
[10] H. Brückmann, *Antennen* (Verlag von S. Hirzel, Leipzig, 1939), p. 113.

an integral and we find

$$G=\frac{1}{2\pi}\int_0^{2\pi} e^{-ik\rho'\cos\beta}d\beta=\frac{1}{\pi}\int_0^{\pi}\cos(k\rho'\cos\beta)d\beta=J_0(k\rho'),$$

where $J_\gamma(z)$ denotes the bessel function of order γ and argument z. This expression will be further discussed later and illustrated by diagrams in the special case where the principal direction is horizontal.

We now return to the case where the number of antennas is finite. For further transformation of the above developed expression for the array characteristic we then use

$$\exp(iz\sin\alpha)=J_0(z)+\sum_{n=1}^{\infty}[\exp(in\alpha)+(-1)^n\exp(-in\alpha)]J_n(z).^{11}$$

Setting

$$\alpha=v-\pi/2$$

we find

$$\begin{aligned}\exp(-iz\cos v)&=J_0(z)+\sum_{n=1}^{\infty}(-i)^n[\exp(inv)+\exp(-inv)]J_n(z)\\&=J_0(z)+2\sum_{n=1}^{\infty}(-i)^n\cos nvJ_n(z).\end{aligned}$$

By using this formula we may express the array characteristic in the following way

$$G=J_0(k\rho')+2\sum_{n=1}^{\infty}(-i)^nJ_n(k\rho')1/s\sum_{j=1}^{s}\cos n(\xi-\beta_j).$$

Recalling the meaning of β_j,

$$\beta_j=j2\pi/s,$$

we may make use of the familiar formula

$$\frac{1}{s}\sum_{j=1}^{s}\cos n\left(\xi-j\frac{2\pi}{s}\right)=\begin{cases}\cos ps\xi \text{ for } n/s=p, \text{ where } p=0,\pm1,\pm2,\cdots,\\ 0 \text{ in any other case.}\end{cases}$$

Substituting in the expression for the array characteristic we obtain

$$G=J_0(k\rho')+2\sum_{p=1}^{\infty}(-i)^{ps}J_{ps}(k\rho')\cos ps\xi.$$

We now have to treat separately the case where the number of antennas is even, and the case where the number is odd.

When s is even:

$$G=J_0(k\rho')+2\sum_{p=1}^{\infty}J_{ps}(k\rho')\cos\left(\frac{\pi}{2}-\xi\right)ps.$$

When s is odd:

$$G=J_0(k\rho')+2\sum_{p=1}^{\infty}J_{2ps}(k\rho')\cos\left(\frac{\pi}{2}-\xi\right)2ps-i2\sum_{p=0}^{\infty}J_{(2p+1)s}(k\rho')\sin\left(\frac{\pi}{2}-\xi\right)(2p+1)s.$$

The first term in the expression for the array characteristic as well in the case of s even as in the case of s odd is seen to be identical with the array characteristic in the case of $s=\infty$. The first term in each of the above expressions may be considered the principal term and the remaining terms correction terms, which express the difference between the array characteristic for the array in question and the array characteristic for the corresponding array with infinitely many elements. Since a bessel function of constant argument decreases rapidly with increasing order, it follows that the array characteristic for an increasing number of antennas, s, converges rapidly to the array characteristic for $s=\infty$ as was expected. As a rule it is desirable to choose s so large that the array characteristic is a good approximation to the array characteristic in the case of $s=\infty$; on the other hand, for practical reasons it is desirable to choose s as small as possible.

The array characteristic in the case of s even is derived by Stenzel approximately in the way described here. The correction terms in the case of s even are seen to be in phase with the principal term. On the other hand, in the case of s odd, there occur correction terms in phase with, as well as correction terms 90° out of phase with, the principal term. Let us for a moment suppose that we have chosen s so large, that the correction terms are small compared with the maximum value of the principal term, i.e., 1. The array characteristic may then sufficiently accurately be expressed by the principal term and the first term in each of the infinite series of correction terms.

When s is even:

$$G=J_0(k\rho')+2J_s(k\rho')\cos(\pi/2-\xi)s,$$

$$|G|=|J_0(k\rho')+2J_s(k\rho')\cos(\pi/2-\xi)s|.$$

When s is odd:

$$G=J_0(k\rho')+2J_{2s}(k\rho')\cos(\pi/2-\xi)2s-i2J_s(k\rho')\sin(\pi/2-\xi)s,$$

$$|G|=([J_0(k\rho')+2J_{2s}(k\rho')\cos(\pi/2-\xi)2s]^2+[2J_s(k\rho')\sin(\pi/2-\xi)s]^2)^{\frac{1}{2}}.$$

Apart from the immediate neighborhood of the zeros

[11] G. N. Watson, *Theory of Bessel Functions* (The Cambridge University Press, Cambridge, England, 1944), p. 22.

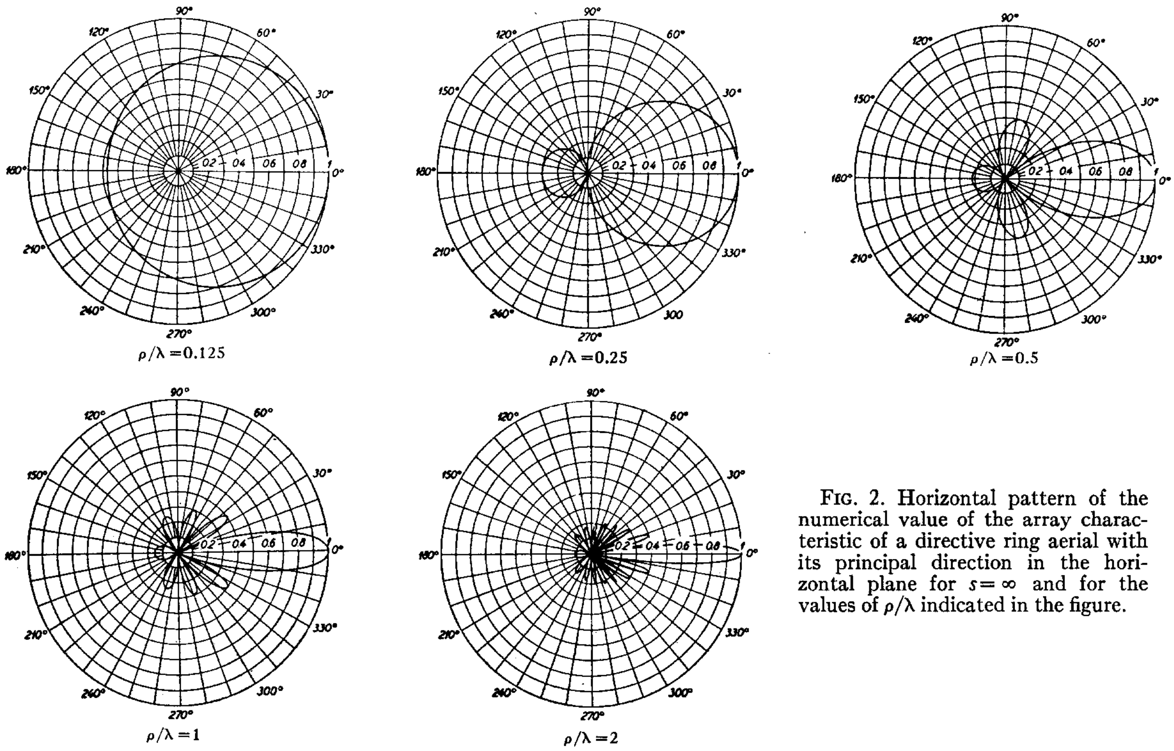

FIG. 2. Horizontal pattern of the numerical value of the array characteristic of a directive ring aerial with its principal direction in the horizontal plane for $s=\infty$ and for the values of ρ/λ indicated in the figure.

of $J_0(k\rho')$ this may be expressed approximately as

$$|G|=|J_0(k\rho')+2J_{2s}(k\rho')\cos(\pi/2-\xi)2s| \\ +(2[J_s(k\rho')\sin(\pi/2-\xi)s]^2)/|J_0(k\rho')|,$$

because a bessel function of constant argument decreases so rapidly with increasing order that the correction term containing the bessel function of order $2s$ may be neglected compared with the correction term containing the bessel function of order s.

Since the correction term 90° out of phase with the principal term is the most important term in the case of s odd, this case will be more favorable than the case of s even as regards the approximation to the array characteristic for $s=\infty$. This will be demonstrated later by a numerical example in the case where the principal direction is horizontal. At first, however, we shall give a short treatment of the case where the principal direction is vertical, thereby establishing the connection between Page's work and the present investigation.

THE PRINCIPAL DIRECTION PERPENDICULAR TO THE PLANE OF THE CIRCLE

For a ring aerial with its principal direction perpendicular to the plane of the circle, i.e., $\theta_0=0$, we find

$$\delta_j=0.$$

The currents in all of the antennas consequently are in phase. By adding to this array an antenna at the center of the circle with a current 180° out of phase with these currents we obtain the fading reducing, concentric ring aerial referred to above.

Setting $\theta_0=0$ in the expressions for ξ and ρ' we get

$$\xi=\varphi,$$

$$\rho'=\rho\sin\theta.$$

Substituting in the expressions for the array characteristic we obtain

when s is even:

$$G=J_0(k\rho\sin\theta)+2\sum_{p=1}^{\infty}J_{ps}(k\rho\sin\theta)\cos\left(\frac{\pi}{2}-\varphi\right)ps;$$

when s is odd:

$$G=J_0(k\rho\sin\theta)+2\sum_{p=1}^{\infty}J_{2ps}(k\rho\sin\theta)\cos\left(\frac{\pi}{2}-\varphi\right)2ps$$

$$-i2\sum_{p=1}^{\infty}J_{(2p+1)s}(k\rho\sin\theta)\sin\left(\frac{\pi}{2}-\theta\right)(2p+1)s.$$

The case of s even was discussed by Chireix;[1] however, besides the principal term of the array characteristic he only gives the first correction term. Page[8] investigated in detail as well the case of s even as the case of s odd and concluded that a much better approximation to the array characteristic in the ideal case of $s=\infty$ is obtained by using an odd number of antennas than by using an even number.

Since the special case mentioned here has been discussed at length in the literature, no further discussion of it will be given here.

THE PRINCIPAL DIRECTION IN THE PLANE OF THE CIRCLE

In this paper we shall especially deal with the case where the principal direction of the antenna array is horizontal, i.e., $\theta_0=\pi/2$. Confining our investigation to the horizontal plane $\theta=\pi/2$ we obtain for this value of θ

$$\xi=\pi/2+(\varphi+\varphi_0)/2),$$
$$\rho'=2\rho\sin((\varphi-\varphi_0)/2).$$

By substituting in the expressions for the array characteristic for a ring aerial with s antennas we get

when s is even:

$$G=J_0(2k\rho\sin((\varphi-\varphi_0)/2))$$
$$+2\sum_{p=1}^{\infty}J_{ps}(2k\rho\sin((\varphi-\varphi_0)/2))\cos[(\varphi+\varphi_0)/2]ps;$$

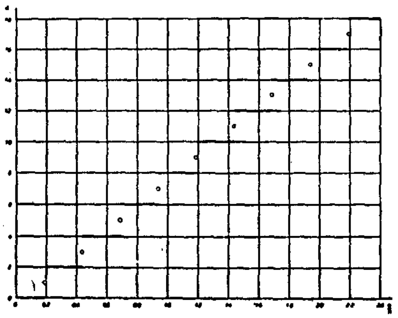

FIG. 3. Number of lobes, n, in the horizontal pattern of the array characteristic of a directive ring aerial with the principal direction in the horizontal plane for $s=\infty$ as a function of ρ/λ.

when s is odd:

$$G=J_0(2k\rho\sin((\varphi-\varphi_0)/2))$$
$$+2\sum_{p=1}^{\infty}J_{2ps}(2k\rho\sin((\varphi-\varphi_0))/2)\cos[(\varphi+\varphi_0)/2]2ps$$
$$+i2\sum_{p=0}^{\infty}J_{(2p+1)s}(2k\rho\sin((\varphi-\varphi_0)/2))$$
$$\times\sin[(\varphi+\varphi_0)/2](2p+1)s.$$

The expression for the array characteristic in the case of s even was derived by Stenzel[9] whereas no discussion of the case of s odd seems to have been published. As was already mentioned and as will be further discussed in what follows, the case of s odd differs essentially from that of s even, the first case being more favorable than the second as was the case with the fading reducing, concentric ring aerial investigated by Page.

According to what was shown above, the principal term $J_0(2k\rho\sin((\varphi-\varphi_0)/2))$ is identical with the array characteristic for the corresponding ring aerial with $s=\infty$, so the remaining terms may be considered correction terms. From the expression for the array characteristic in the case of $s=\infty$, for directions in the horizontal plane, we see that it is a function only of the angle v between the direction φ and the principal direction φ_0, $v=\varphi-\varphi_0$. Consequently, when the principal direction is rotated in the horizontal plane an angle $\Delta\varphi_0$ by readjusting the current phases in the way described above, the whole horizontal pattern will be rotated the angle $\Delta\varphi_0$ with its shape unchanged.

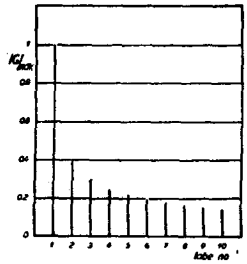

FIG. 4. The maximum value of the fully developed lobes in the horizontal pattern of the array characteristic of a directive ring aerial with the principal direction in the horizontal plane for $s=\infty$.

In Fig. 2 the horizontal pattern of the numerical value of the array characteristic is plotted for $s=\infty$ corresponding to various values of the ratio of the radius ρ of the circle to the wavelength λ. Several of these diagrams are calculated and plotted in the paper by Stenzel[9] referred to above; they are included here for the sake of completeness. From the expression for the array characteristic as well as from the diagrams in Fig. 2 it is seen that the horizontal pattern is symmetrical with respect to $v=0$ and that the number of lobes increases with increasing radius of the circle, whereas the width of the principal lobe decreases. In Fig. 3 the number of lobes, n, is plotted as a function of the ratio of the radius of the circle, ρ, to the wavelength λ. The maximum value of the fully developed lobes in the half-plane $0<v<\pi$ is shown in Fig. 4, where the principal lobe is denoted as number 1.

The width of the principal lobe may be measured by the angle α between the two zero-directions confining this lobe or better by the angle β between the two directions for which the array characteristic assumes $1/\sqrt{2}$ of its maximum value. The angle α is determined as the

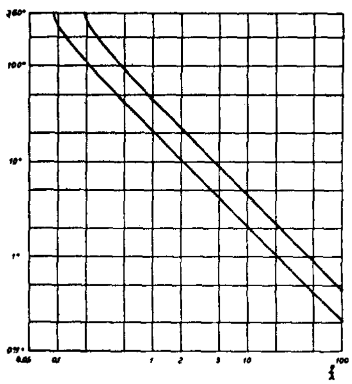

FIG. 5. The angle α between the two zero directions confining the principal lobe in the horizontal pattern of the array characteristic of a directive ring aerial with the principal direction in the horizontal plane for $s=\infty$, and the angle β between the two directions for which the array characteristic is $1/\sqrt{2}$ of its maximum value, plotted as functions of ρ/λ.

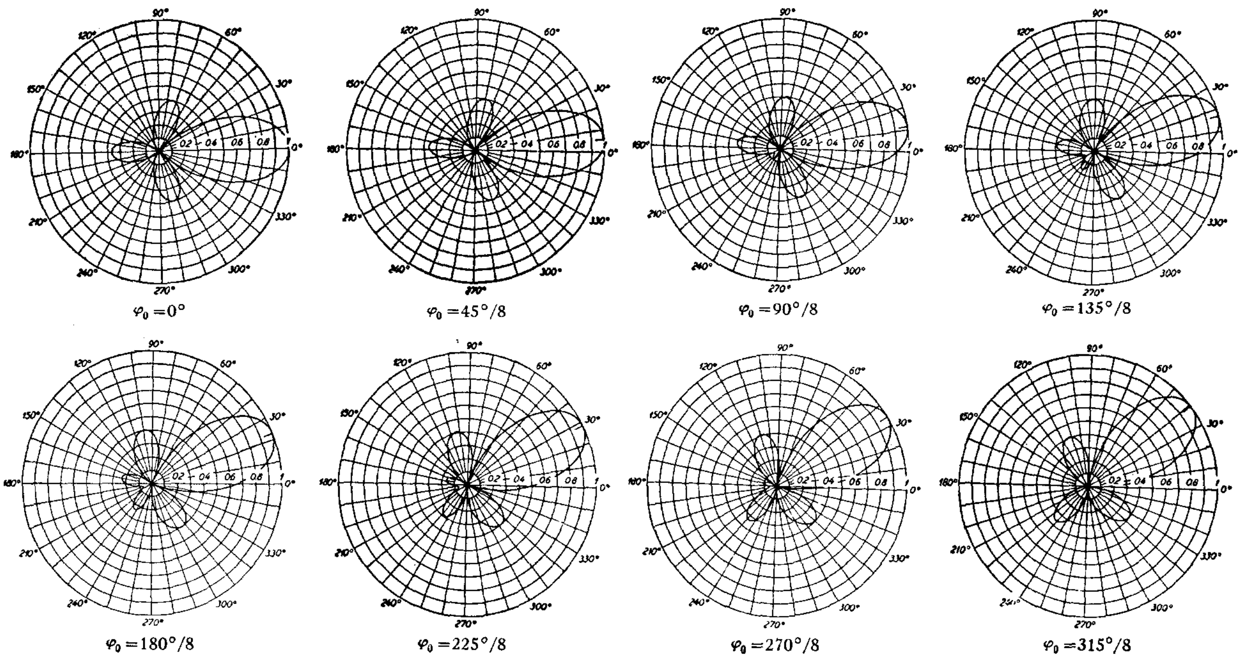

FIG. 6. The horizontal pattern of the array characteristic of a directive ring aerial with the principal direction in the horizontal plane and with $\rho/\lambda=0.5$ and $s=8$ for the values of the principal direction φ_0 indicated in the figure. The position of the antenna system is fixed.

smallest root in the equation

$$J_0(2k\rho\sin(\alpha/4))=0,$$

whereas the angle β is determined as the smallest root in the equation

$$J_0(2k\rho\sin(\beta/4))=1/\sqrt{2}.$$

The angles α and β are plotted in Fig. 5 as functions of ρ/λ.

Let us now return to the general case where the number of antennas, s, is finite. Since the array characteristic for horizontal directions, as appears from the expressions derived above, is a function of not only $\varphi-\varphi_0$ but also $\varphi+\varphi_0$, the horizontal pattern will change its shape when the principal direction is rotated by adjusting the current phases in the way prescribed above. From the above derived expressions it is seen that the absolute value of the array characteristic, $|G|$, in the case of s even will have the period in φ_0, $2\pi/s$, whereas in the case of s odd it will have the period π/s. Consequently, we need only investigate $|G|$ for a variation of φ_0 in an interval of the length $2\pi/s$ and π/s for s even and s odd respectively.

NUMERICAL EXAMPLE

The theory developed above will be illustrated by a numerical example. We consider a ring aerial with the principal direction in the horizontal plane and with $\rho/\lambda=0.5$. For eight antennas in the array, i.e., for $s=8$, and for the principal directions $\varphi_0=0$, $45°/8$, $90°/8$, $\cdots$, $315°/8$ the horizontal pattern of the array characteristic is plotted in Fig. 6. In these diagrams the antenna array is fixed whereas the principal direction rotates as indicated. The figure shows the continuous deformation of the horizontal pattern caused by the rotation of the principal direction. The period is seen to be $360°/8=45°$ in accordance with the theory.

The horizontal pattern of the absolute value of the array characteristic is calculated for $s=5$, 6, 7, 8, and 9. In the case of s even the pattern is calculated for $\varphi_0=0$, $90°/s$, $180°/s$, and $270°/s$, and in the case of s odd for $\varphi_0=0$ and $90°/s$, the pattern for $\varphi_0=180°/s$ in this case being identical with that for $\varphi_0=0$ and the pattern for $\varphi_0=270°/s$ identical with that for $\varphi_0=90°/s$. The results are plotted in Fig. 7 where the horizontal pattern of the absolute value of the array characteristic is plotted as a function of the angle, v, between the direction φ and the principal direction φ_0. Consequently, in the diagrams in Fig. 7 the principal direction is fixed, whereas the array rotates; this way of presenting the results makes it easier to compare the various diagrams.

It is impossible to account for the difference between a horizontal pattern corresponding to certain values of s and φ_0 and the horizontal pattern corresponding to $s=\infty$ by a single number. But a consideration of Fig. 7 gives the following result. In the case of only five antennas, the deviations of the horizontal pattern from the pattern corresponding to $s=\infty$ are so large that the minor lobes are of the same order of magnitude as the principal lobe. No improvement is obtained by increasing the number of antennas to six, for the deviations from the ideal pattern are of approximately the same magnitude as in the case of only five antennas.

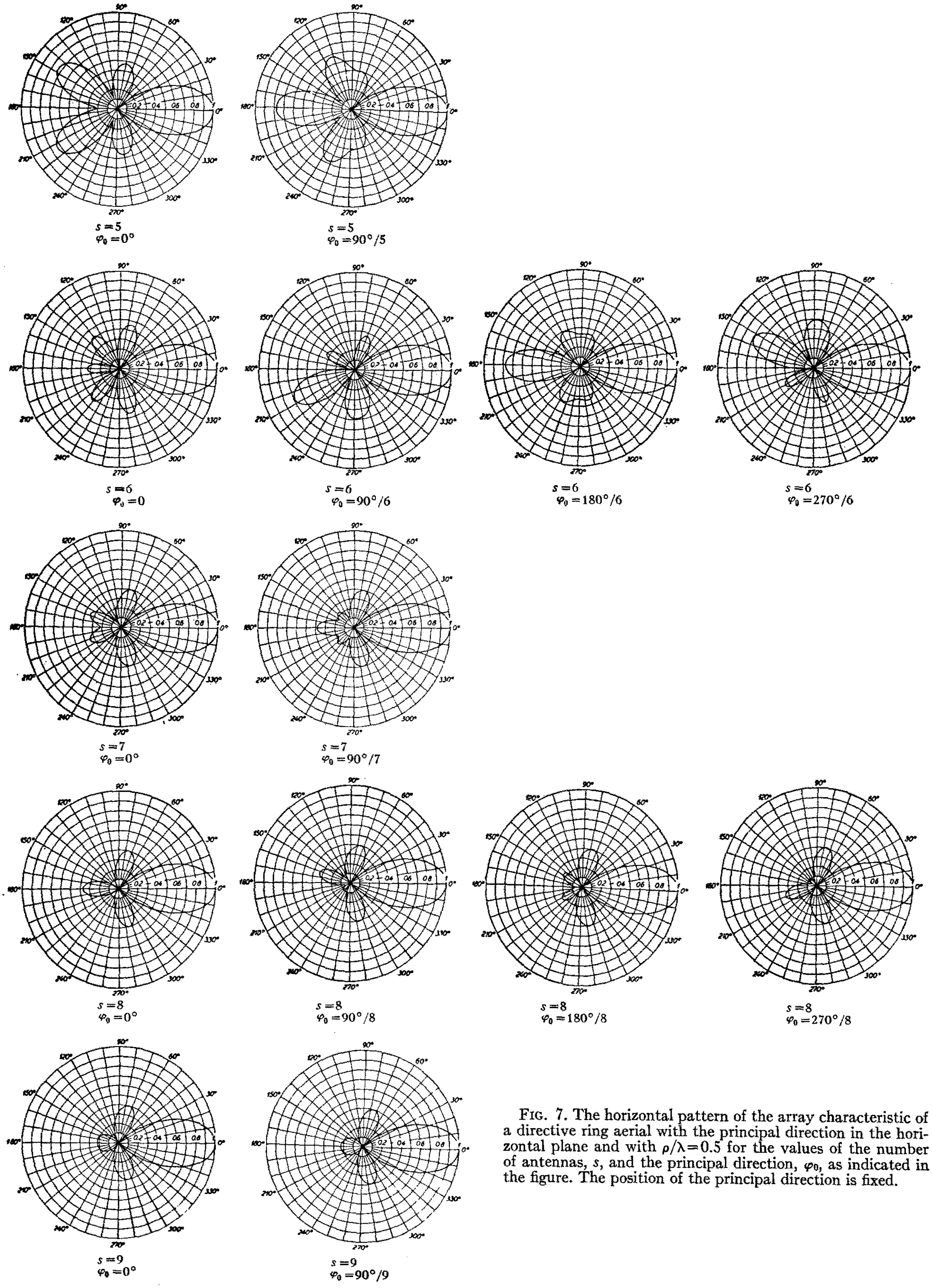

FIG. 7. The horizontal pattern of the array characteristic of a directive ring aerial with the principal direction in the horizontal plane and with $\rho/\lambda=0.5$ for the values of the number of antennas, s, and the principal direction, φ_0, as indicated in the figure. The position of the principal direction is fixed.

When the number of antennas is increased from six to seven the situation is improved essentially, for the minor lobes now are essentially smaller than the principal lobe. In the backward direction, however, the horizontal pattern diverges considerably from the ideal pattern. The horizontal patterns corresponding to eight antennas do not approximate the ideal pattern better than do the patterns corresponding to seven antennas. However, an improvement is obtained again when the number of antennas is increased from eight to nine. The deviations of the horizontal patterns corresponding to nine antennas from the ideal pattern are of the same order of magnitude as the accuracy with which the diagrams are plotted. A similar result is found in the case of ten antennas; but the patterns obtained in this case are not plotted in the figure. For a still larger number of antennas the horizontal patterns practically coincide with the pattern corresponding to infinitely many antennas. The cases considered here are seen to group so that the cases $s=5$ and $s=6$ approximate the ideal pattern with the same accuracy and that the same statement applies to the cases $s=7$ and $s=8$ and to the cases $s=9$ and $s=10$, whereas the accuracy changes considerably when we pass from one group to the next one.

The numerical example thus confirms that a ring aerial of the type considered here and with an odd number of elements has a marked advantage over a similar ring aerial with an even number of elements.

Stenzel[9] has shown that the side lobes may be considerably reduced by adding to the ring aerial in question one or more ring aerials concentric with the first one. It would be interesting to make an investigation concerning the influence of the number of elements on the horizontal pattern of this more complex antenna system similar to what has been made here for the simple ring aerial. But this investigation is reserved for another paper.

CONCLUSION

The general expression is derived for the array characteristic of a ring aerial with an arbitrarily chosen principal direction and with a finite number of elements. This expression shows that no uniform improvement of the approximation of the array characteristic to the ideal characteristic is obtained when the number of elements is increased, but that ring aerials with an odd number of elements are more favorable in this respect than aerials with an even number. This was shown by Page[8] for the special case where the principal direction is perpendicular to the plane of the circle. The case where the principal direction lies in the horizontal plane is especially investigated here. A numerical example confirms the above statement concerning the superiority of an odd number of antennas over an even number.

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